RESEARCH ARTICLE

Application of the Function in Basic Mathematics

Laxmi Rathour[1], Dragan Obradovic[2], Lakshmi Narayan Mishra[3,*]
Kejal Khatri[4*], Vishnu Narayan Mishra[5]

[1]Ward Number -16, Bhagatbandh, Anuppur 484 224, Madhya Pradesh, India
[2]Department of Mathematics and Informatics, School "Agricultural High School" Pozarevac – Serbia
[3]Department of Mathematics, School of Advanced Sciences, Vellore Institute of Technology, Vellore 632 014, Tamil Nadu, India
[4*]Department of Mathematics, Government College Simalwara, Dungarpur 314403, Rajasthan, India
[5]Department of Mathematics, Indira Gandhi National Tribal University, Lalpur, Amarkantak 484 887, Madhya Pradesh, India

ABSTRACT: The concept of function is one of the fundamental mathematical concepts, very important within mathematics itself as well as in the application of mathematics. Functions are an essential element of mathematical structuring and modeling of problems (e.g.in algebraic structures), as well as a means of comparing structures thus obtained (eg homomorphisms of structures). A mathematical function is a rule that gives the value of the dependent variable corresponding to certain values of one or more independent variables. A function can be represented in several ways, such as a table, formula, or graph. Apart from isolated points, the mathematical functions found in physical chemistry are single-valued. Apart from isolated points, the mathematical functions that occur in physical chemistry are continuous.

KEYWORDS: mathematical function, derivative of functions, use of derivatives

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1.INTRODUCTION

Calculus is one of the most important branches of mathematics that deals with continuous changes. The two main concepts on which calculus is based are derivatives and integrals. The derivative of a function is a measure of the rate of change of the function, while the integral is a measure of the area under the curve of the function. The derivative gives an explanation of the function at a particular point, while the integral accumulates the discrete values of the function over a range of values.

Elements of mathematical analysis occupy an important place in the school mathematics course. Students are introduced to the mathematical apparatus that can be used effectively in solving many problems in mathematics, physics and technology. With the help of differential calculus, the properties of functions are studied, their graphs are constructed and problems for the largest and smallest value are solved. In other words, the introduction of a new mathematical apparatus makes it possible to consider a number of problems that could not previously be solved by elementary methods. At the same time, the derivative can be used to solve elementary mathematical problems.

This use of derivatives is very useful for two reasons. On the one hand, many traditional problems of elementary mathematics (proof of inequalities, identities, investigation and solution of equations and their systems) are efficiently solved using derivatives. On the other hand, the non-standard use of elements of mathematical analysis allows for a better understanding of the basic terms of the theory being studied, because one should choose a method for solving the task, check the conditions of its applicability and analyze the obtained results. In addition, the methods of mathematical analysis can be used not only to solve problems, but also be a source for obtaining new facts of elementary mathematics [1-6].

Corresponding Author: kejal0909@gmail.com, lakshminarayanmishra04@gmail.com
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The use of derivatives in elementary mathematics is quite varied. First of all, these are tasks in the process of solving which it becomes necessary to investigate the function using derivatives and build its graph. This use of the derivative is the most traditional in the school [7-10].

The derivative of a function is the function itself, so we can find the derivative of the derivative. For example, the derivative of the position function is the rate of change of position or speed. The derivative of velocity is the rate of change of velocity, which is acceleration. The new function obtained by differentiating the derivative is called the second derivative. Furthermore, we can continue to take derivatives to get the third derivative, the fourth derivative, etc. Together, they are called higher-order derivatives. Notation for higher-order derivatives of i=f(k).

Mathematicians are often interested in general observations, say describing patterns that apply in a large number of cases. Think about the Pythagorean Theorem: it doesn't tell us something about a single right triangle, but about every right triangle. People who use applied mathematics, such as engineers and economists, often encounter the same types of functions where only small changes in certain constants occur. These constants are called parameters.

This means that when the derivative changes sign from positive to negative or negative to positive there must be a turning point or peak on the graph. These are called maximum and minimum points. They are not necessarily the largest or smallest total values of the function.

Remember, a continuous function cannot change between negative and positive values without going through zero, so we look for stationary points by finding out when the derivative is 0.

2. DERIVATIVE OF THE FUNCTION

The need for the derivative of the function arose when trying to find a universal way to determine the tangent of a curve in geometry and the speed of movement in mechanics.

The concept of the derivative of a function is one of the basic concepts of mathematical analysis. It is known that the concept of derivative arose from the need to correctly introduce concepts such as tangent to a curved line, i.e. speed and acceleration. Today, the concept of derivative and concepts that are a generalization of the classical concept of derivative have a much wider application both in mathematics and natural sciences, as well as, for example, in economics. In high school teaching, the concept of derivation is extremely suitable to make a connection between two subjects - mathematics and physics. Practice shows that this connection is not established at all (the study of derivatives is reduced to working with a table of derivatives) or is not established in an adequate way (for example, the difference between the concept of velocity as a vector (eng. velocity) and the concept of speed as the intensity of a vector is not clarified (Eng. speed)). We can see the application of the derivation in physics when considering the dynamics of a material point (eg in free fall, downward shot, upward shot, oblique and horizontal shot). For a given function, it is of particular interest to study the properties of the derivative of that function, because this allows us to find extrema and examine the convexity of the given function [11,12].

2.1. Approaches to processing the concept of the derivative of a function

Calculus, and especially its part related to the derivative of a function, has always been one of the most challenging areas of mathematics, both for teaching and learning, due to its dynamic nature and complex concepts. Methodological approaches used to introduce and process the concept of the derivative of a function have always been the subject of controversy due to the specific nature of the derivative, which is reflected in several different representations (algebraic, graphic, and numerical) and interpretations (dynamic, geometric). The existence of multiple representations in students leads to difficulties in learning and understanding the derivation of a function, and later to difficult application to solving problems from real practice.

Students' understanding of differentiation was Orton [14], who found that students easily mastered the skills of calculating derivatives, but did not understand the relationship between increments and the graphical representation of derivatives. Analysis of the answers to the differentiation and rate of change tasks led to detailed data on the level of understanding achieved and common errors and misconceptions.

Students tendency to assume similarities between a function and its derivative

Nemirovsky and Rubin [11]. Aspinwall et al., [12] showed that even when students understand the derivative as the slope of a tangent (geometric interpretation) sketching the graph of the derivative function based on the graph of the original function can be very challenging for them. "A student who understands the concept of derivative can explain how the average ratio of function increments and argument increments approaches their current ratio, and how the slopes of the intercepts approach the slope of the tangent by applying the concept of limit value.". For the above reasons, it can be concluded that for understanding the derivative of a function it is particularly important to emphasize its meaning, i.e. to give students the opportunity to familiarize themselves with its various representations and ways of application.

3. THE TERM OF THE DERIVATIVE OF A FUNCTION

Derivatives are fundamental to solving problems in calculus and differential equations. In general, scientists observe changing systems (dynamical systems) to obtain the rate of change of some variable of interest, incorporate this information into some differential equation, and use integration techniques to obtain a function that can be used to predict the behavior of the original system under various conditions.

Geometrically, the derivative of a function can be interpreted as the slope of the graph of the function or, more precisely, as the slope of the tangent line at a point. Its calculation, in fact, derives from the slope formula for a straight line, except that a limiting process must be used for curves. The slope is often expressed as the “rise” over the “run,” or, in Cartesian terms, the ratio of the change in y to the change in x. This change in notation is useful for advancing from the idea of the slope of a line to the more general concept of the derivative of a function.
By deliberately discussing the tangent and the mean and instantaneous speed, i.e. the coefficient of the direction of the tangent, we came to the concept of the derivative:

$$y' = f'(x)$$

As with the definition of the current speed, as far as the law of the road is known *s = f(t)*,to the term derivative we can arrive by any calculation of the rate of change of some quantity in the course of time if the law of the dependence of that quantity on time is known.

Definition

Derivation of function y = f (x), per argument x is the limit value of the quotient of the increment of the function and the increment of the argument when the increment tends to zero, i.e.

$$y' = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x} = f'(x)$$

When we talk about derivatives, a 3-letter word is often mentioned - differentiation. Differentiation is nothing but a limiting process by which the derivative of the y' function *y* is arrived at. A function *y = f (x),* that has a derivative *x* at a point is said to be differentiable at that point. When we say that a function is differentiable on an interval,*(a,b)* it means that it is differentiable at every point of the interval.
We saw even before we defined the derivation that it (as was said at the beginning) has a great application.

Let us consider one important property of the derivative of a function, which is differentiability and continuity. Before we interpreted the derivative, we assumed that our function must be continuous. Continuity and differentiability form the following theorem:

Theorem 3.1

If the function *y = f (x),* defined on the interval *(a,b)* has a derivative at a point belonging to that interval, that is $x \in (a,b)$, (that is, it is differentiable at a given point), then it is also continuous.

Proof:
The assumption of the theorem is that the function is differentiable at a point *x* i.e. exists $\lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x}$. If we have $\Delta x \neq 0$, then we can write

$$\Delta y = \Delta y \cdot \frac{\Delta x}{\Delta x}$$

Now we have, if we apply the limit process to the last expression:

$$\lim_{\Delta x \to 0} \Delta y = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} \cdot \lim_{\Delta x \to 0} \Delta x = 0$$

So when $\Delta x \to 0$,then $\Delta y \to 0$. This means that the differentiable function $y = f(x)$ is at the same time continuous at a given point. This is one of the most important theorems related to the function of the function. Without this theorem, we could not easily "walk" the area of the excerpt. Almost at any task concerned with the excerpts of a function, this theorem is used. If you wondering if the reverse theorem is valid, i.e., whether the differential function is uninterrupted, the answer to this question would be "No".

Theorem 3.2

Proof:

We will prove this theorem by stating only one example that it says that the turn is not valid. Let's watch the function $y = f(x)$. This function is constantly at the entire interval of real numbers. From the image it can be seen that $\lim_{\Delta x \to -0} \frac{\Delta y}{\Delta x} = -1$ a $\lim_{\Delta x \to +0} \frac{\Delta y}{\Delta x} = 1$. From the recent terms we see that the limit value of the amount is $\frac{\Delta y}{\Delta x}$ for the left and right limit value per argument Δx different, which means that the derivation of the function y = ▯ x▯ in the point (0,0) there is no unique statement. In other words of functions $y = f(x)$ in the point (0,0) is not differential. Proof theorems are completed.

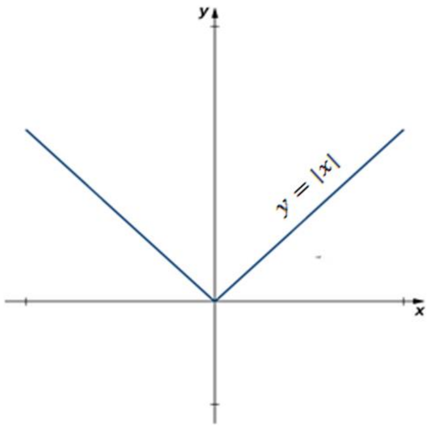

Figure 1: Graph of the function y = ▯ x▯

3.1. Geometric interpretation of the derivative of a function

Figure 2 shows a fragment of the graph of the function$y = f(x)$.

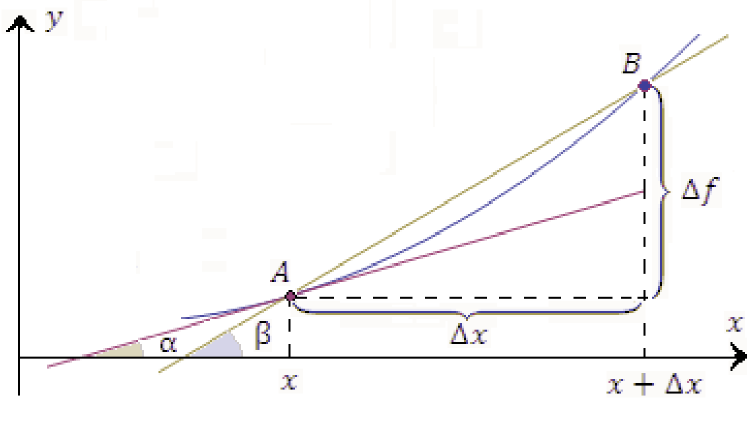

Figure 2: shows a fragment of the graph of the function

Display 1. Secant AB forms an angle β with the positive direction of the axis *0 x.* The tangent to the graph of the function is drawn at point *A.*

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The slope of the secant AB is equal to the average rate of change of the function *f (x)* on the interval [*x, x + Δ x*]:

$$\mathrm{tg}\,\beta = \frac{f(x+\Delta x)-f(x)}{\Delta x} = \frac{\Delta f}{\Delta x}.$$

The limit position of the secant *AB* when moving point *B* to point A along the arc of the curve *y = f(x)* is the tangent to the graph at point *A*. Therefore, the slope of the tangent is equal to the limit of the slope of the secant as *Δ x → 0*:

$$\mathrm{tg}\,\alpha = \lim_{\Delta x\to 0} \mathrm{tg}\,\beta = \lim_{\Delta x\to 0}\frac{f(x+\Delta x)-f(x)}{\Delta x} = \frac{df}{dx}$$

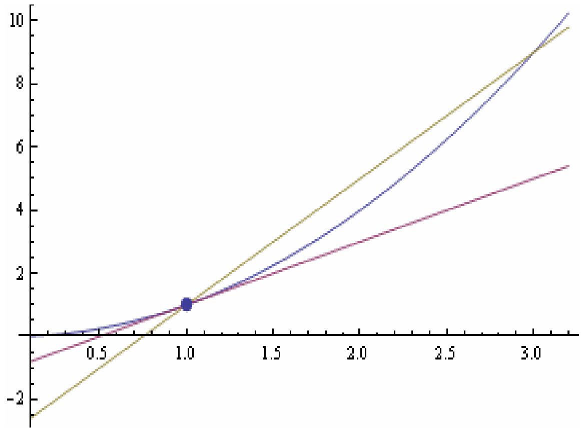

Figure 3: The tangent is the limiting position of the secant

Display 2 The tangent is the limit position of the secant *AB* when moving point *B* to point *A*.
Therefore, the derivative of *f' (x)* at point *x* is equal to the tangent of the angle formed by the tangent to the graph of the function*y = f (x)* at this point with the positive direction of the axis *0 x.*

4. IMPORTANCE OF THE FUNCTION FOR THE FIELD OF MATHEMATICS

Today, mathematics is no longer as exclusively related to the physical sciences as it was in the past. It experienced a significant increase in application domains, becoming an instrument for studying phenomena and situations in biological sciences, human and social sciences, business, communications, engineering and technology. Mathematics represents the essential means of description, explanation, prediction and control. All applications of mathematics presuppose the notion of a model. A mathematical model is a representation through relationships and structures that intends to describe the found fundamental elements in a given situation while deliberately omitting secondary elements. And a mathematical model can take several forms, but it is usually made up of variables, the relationships between those variables, and their corresponding rates of change [11,15,7,8].
Today, we understand functions as relations in which the value of a variable depends on one or more other variables. Special values of the independent variable generate one and only one value of the dependent variable. Learning about functions and using them as tools to model situations is a long process that starts in elementary school with simple equations and elementary data representation and using one-to-many, one-to-one, and many-to-one mappings in different contexts. Students in high school need to coordinate algebraic, graphical and numerical data, often from scientific experiments or statistical investigations. Trigonometry uses algebraic manipulation to combine geometric concepts and angle measurement. These important ideas are highlighted by Watson, Jones and Pratt [16]. Many relevant original texts with their translations are available in Stedall (2008) [17].
Using individual data points from physical experiments to investigate a possible relationship between a process and its outcome raises questions about how to represent and the meaning of a line that can be drawn to connect an individual point on a graph. Knowledge of the history of mathematics shows us how the

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modern sophisticated concept of a function grew out of the vital contributions of many mathematicians and scientists.

4. CONCLUSION

Historically speaking, the concept of derivative arose, on the one hand, by considering the physical problem of determining the current speed of movement of a body, and on the other hand, by considering the mathematical problem of determining the coefficient of the direction of the tangent of some curve. That is why the term derivative is a suitable term that we can use to describe the connection between mathematics and physics, especially to students.

The derivative of a function is one of the basic concepts of mathematics. Together with the integral, derivative occupies a central place in calculus. The process of finding the derivative is called differentiation. The inverse operation for differentiation is called integration. The derivative of a function at some point characterizes the rate of change of the function at this point. We can estimate the rate of change by calculating the ratio of change of the function $\Delta y$to the change of the independent variable Δx. In the definition of derivative, this ratio is considered in the limit as $\Delta x \rightarrow 0$.A function denoting the rate of change of another function is called as a derivative of that function. In other words, a derivative is used to define the rate of change of a function.

As we have seen, the derivative of a function at a given point gives us the rate of change or slope of the tangent line to the function at that point. If we differentiate the position function at a given instant, we get the velocity at that instant. It seems reasonable to conclude that knowing the derivative of the function at each point would produce valuable information about the behavior of the function.

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